Fivebrane Lagrangian with Loop Corrections in Field-Theory Limit [1]

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Abstract

Equations of motion and the lagrangian are derived explicitely for Dual D=10, N=1 Supergravity considered as a field theory limit of a Fivebrane. It is used the mass-shell solution of Heterotic String Bianchi Identites obtained in the 2-dimensional σ-model two-loop approximation and in the tree-level Heterotic String approximation. As a result the Dual Supergravity lagrangian is derived in the one-loop Five-Brane approximation and in the lowest 6-dimensional σ-model approximaton.

1 Introduction

There are two kinds of parameters, characterising the heterotic superstring field-theory limit. The *first parameter* is the string-tension α' which enters into the σ-model lagrangian, corresponding to the superstring tree-level:

$$L_{HS} = -\frac{1}{2\pi\alpha'} \int d^2\xi \left(\frac{1}{2}\sqrt{-\gamma}\gamma^{ij}\partial_i X^m \partial_j X^n g_{mn}^{HS}(X) + \right.$$
$$\left. +\frac{1}{2}\epsilon^{ij}\partial_i X^m \partial_j X^n B_{mn}(X) + \ldots \right) \tag{1}$$

Here α' is the σ-model loop expansion parameter, $R^{(2)}$-is the 2-dimensional curvature.

The Type I supergravity lagrangian that follows from (1) in the field-theory limit takes the form:

$$L_{SG} = \frac{\sqrt{-g^{HS}}}{2k^2} \exp(-2\varphi)(R + 8(\partial_n\varphi)^2 + \frac{1}{12}K_{mnp}^2 + \ldots) \tag{2}$$

Here k^2 is the Newton gravitational constant, φ is the dilatonic field, R is the curvature scalar. All the entries in (2) are calculated in terms of string metric: $g_{mn}^{HS} = exp(\varphi/2)g_{mn}$, where g_{mn} is a canonical metric tensor; K is the 3-form axionic gauge-field, corresponding to the 2-form potential B in (1):

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$$K = dB + 2\alpha'(\Omega_G - \Omega_L) \tag{3}$$

where Ω_G and Ω_L are Chern-Symons (CS) 3-form terms, corresponding to the gauge-group (G) and Lorentz $O(1,9)$ group (L).

The *second parameter* of the Heterotic String theory is $\sim k^2/\alpha'$. It is the string loop expansion parameter. The general form of the supergravity lagrangian in the superstring field-theory limit takes the form in terms of string metric [1]:

$$L_{SG} = \frac{\sqrt{-g^{HS}}}{2k^2} \exp(-2\varphi) \sum_{l=1,2\ldots} \sum_{L=0,1,\ldots} L_{L,l-1}$$

$$L_{L,l-1} = a_{L,l-1}(\alpha')^{l-1} \left(\frac{2\,k^2}{\exp(-2\varphi)(2\pi)^5\alpha'} \right)^L R^{l+3L} + \ldots \tag{4}$$

Here L -is a number of string loops, l is a number of σ-model loops. The value $L = 0$ corresponds to the (classical) heterotic string tree-level.

We keep only terms with curvature in (4). Terms with the gauge-field appear in a series of similar structure. To obtain the generic structure of all the terms one must keep in mind the dimensions (in mass units): $dim(\alpha') = -2$, $dim(k^2) = -8$.

Note that terms, which are presented in (2) correspond to all the bosonic contributions at $l = 1$ and $L = 0$ level. The CS-terms in (3) take into account the (anomaly cancelling) $l = 2$ contribution [2].

Now we turn to the fivebrane. In the σ-model limit it is described by the lagrangian:

$$L_{FB} = -\frac{1}{(2\pi)^3\beta'} \int d^6\xi \left(\frac{1}{2}\sqrt{-\gamma}\gamma^{ij}\partial_i X^m \partial_j X^n g_{mn}^{FB}(X) + \right.$$

$$\left. +\frac{1}{6}\epsilon^{j_1\ldots j_6}\partial_{j_1}X^{m_1}\ldots\partial_{j_6}X^{m_6}C_{m_1\ldots m_6}(X) + \ldots \right) \tag{5}$$

Here β' is the fivebrane σ-model loop expansion parameter. It is expected, that the dual supergravity lagrangian that follows from (5) is dual to (2) and takes the form:

$$L_{DSG} = \frac{\sqrt{-g^{FB}}}{2k^2} \exp(2\varphi/3)(R + \frac{1}{2\cdot 7!}M^2_{n_1\ldots n_7} + \ldots) \tag{6}$$

Here M is the 7-form gauge field, corresponding to the 6-form potentials C in (5) and dual to K. All the entities in (6) are calculated in terms of the fivebrane metric: $g_{mn}^{FB} = \exp(-\varphi/6)g_{mn}$. The M-field is defined by:

$$M = dC + \beta'\,X_7 \tag{7}$$

Here X_7 is the Chern-Simons 7-form, containing curvatures and connections of G and $O(1.9)$-groups [3], [4].

The general structure of dual supergravity lagrangian takes the form [1]:

$$L_{DSG} = \frac{\sqrt{-g^{FB}}}{2k^2} \exp(2\varphi/3) \sum_{l'=1,2\ldots} \sum_{L'=0,1,\ldots} L'_{L',l'-1}$$

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$$L'_{L',l'-1} = a'_{L',l'-1}(\beta')^{l'-1}\left(\frac{2\,k^2}{\exp(2\varphi/3)(2\pi)^5\beta'}\right)^{L'} R^{L'+3l'-2} + \dots \tag{8}$$

Here $2\,k^2/\exp(2\varphi/3)(2\pi)^5\beta'$ is the fivebrane loop expansion parameter, but l' and L' are interpreted as numbers of 6-dimensional σ-model loops and fivebrane loops respectively. The value $L' = 0$ corresponds to the (classical) fivebrane tree-level.

In (6) all the terms are kept, that correspond to the $l' = 1$, $L' = 0$ level. But the CS-terms in (7) take into account $l' = 2$, $L' = 0$ contribution.

The discovery of the heterotic string - fivebrane duality [5], [6], [7] leads presumably to the equality of two series (4) and (8). There is the relation between expansion parameters in these theories [1]:

$$\beta' = \frac{2k^2}{(2\pi^5)\alpha'} \tag{9}$$

This relation makes it possible to establish term-by-term correspondence between (4) and (8). That leads to the statement:

$$a_{Ll} = a'_{L'l'}, \quad L' = l - 1, \quad L = l' - 1 \tag{10}$$

It means that σ-model loop expansion in the heterotic string case reproduces the loop expansion in the fivebrane theory; the loop expansion in the heterotic string case reproduces the σ-model expansion in the fivebrane theory.

Then the String/Fivebrane symmetric form of effective lagrangian follows from (9) and (10) (in terms of canonic metric):

$$L_{S/FB} = \frac{\sqrt{-g}}{2k^2} \sum_{L=0}^{\infty} \sum_{L'=0}^{\infty} a_{LL'}(\alpha')^{L'}(\beta')^L \exp(\varphi(L - L')/2)R^{3L+L'+1} + \dots \tag{11}$$

It is not simple to check independently this beautiful statement, because fivebrane theory has not quantized. Moreover, even the consisent supersymmetric fivebrane lagrangian has not yet constructed.

We hope, that the supersymmetry might present additional insight to the problem. We calculate in the present paper, accepting the picture described above, the supersymmetric one-loop corrections to the dual supergravity lagrangian from the second-loop σ-model corrections in the Type I D=10, N=1 supergravity, considered as the field-theory limit of heterotic superstring.

We get: 1) the supersymmetry transformations, 2) the explicitely supersymmetric equations of motion, and 3) the supersymmetric lagrangian for the fivebrane at the $l' = 1$, $L' = 1$ level in the field-theory limit (that corresponds to the dual supergravity with specific loop corrections). [5]

It is interesting in the framework of described approach to obtain all the one-loop supersymmetric heterotic string corrections from the calculation of $\sim \beta'$ terms in the dual supergravity. It is a problem for the future.

The dual supergravity lagrangian obtained in the present paper corresponds to the supersymmetrised version of $\sim \alpha'$ anomaly cancelling Green-Schwarz (GS) corrections

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[5]As for lagrangian, we are able to present all the bosonic terms. The fermionic terms are too complicated to be presented explicitly. It is a technical problem because one can construct any desired term using the described procedure

[2] to the simple (lowest α'-order) D=10, N=1 supergravity considered in [8]. (That corresponds to the $l = 2$, $L = 0$ level of heterotic string expansion). The problem with the standard supergravity is that $\sim \alpha'$ corrections can be made supersymmetric only in the same $\sim \alpha'$ order. For complete supersymmetrisation one must take into account the infinite number of terms $\sim \alpha'^n$, $(n = 1, 2, \ldots)$, containing the axionic field and dictated by supersymmetry. Situation is different in the dual supergravity. If the same corrections are expressed in terms of fivebrane variables - the result becomes exactly supersymmetric in the order $\sim \alpha'$, i.e. the infinite series in α' is transformed to the finite number of terms in the case of dual supergravity. That means at least that the relation (10) can not be applied directly to terms containing axionic field.

The supersymmetric completion of the standard supergravity (Type I SUGRA) with the GS correction $\sim \alpha'$ has been realized at the mass shell in papers [9], [10], [11] (see [12] for more complete list of references). The lagrangian has not been constructed but it becomes clear, that it contains terms $\sim R^2$ and an infinite number of terms $\sim \alpha'^q K^p$, $q \geq 1$, $p \geq 3$. Several terms of lowest order were found in [13].

The connection between standard and dual supergravity in the superspace approach was mentioned in [14] where explicit calculations were not presented. The iterative scheme for dual transformation in the component approach was suggested in [15].

Few words about notations. (See Appendix 1 for details). This study is made in framework of the superspace approach and our notations in general correspond to [16] (with the change of overall sign in metric signature). The short version of this study was presented in [17] and notations here correspond in general to that paper (small differences are self-evident or explained in the text).

We use the computer program "GRAMA" [18] written in MATHEMATICA for analytical calculations in supergravity.

2 Gravity Sector

We start from the derivation of geometrical equations of motion which are applied equally for usual and dual formulation of supergravity at least for the case $\alpha' \neq 0$, $\beta' = 0$, which is the main approximation accepted in the following. We use:

1) Geometrical Bianchi Identities (BI's) for the supertorsion $T_{BC}{}^D$:

$$D_{[A}T_{BC)}{}^D + T_{[AB}{}^Q T_{|Q|C)}{}^D - \mathcal{R}_{[ABC)}{}^D = 0. \tag{12}$$

Here and in the following $\mathcal{R}_{...}$ means the supercovariant curvature (calculated with the torsion full spin-connection).

2) The set of constraints [19], [20]:

$$T_{\alpha\beta}{}^c = \Gamma_{\alpha\beta}^c \,, \quad T_{a\beta}{}^\gamma = \frac{1}{72}(\hat{X}\Gamma_a)_\beta{}^\gamma \,, \tag{13}$$

where $\hat{X} \equiv X_{abc}\Gamma^{abc}$. The other nonzero torsion components are: $T_{abc} \equiv \eta_{cd}T_{ab}{}^d$ (here T_{abc} is a completely antisymmetric tensor) and $T_{ab}{}^\gamma$. Furthemore $X_{abc} = T_{abc}/72$ as it follows from (12).

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2) Commutation relations for supercovariant derivatives D_A:

$$(D_A D_B - (-1)^{ab} D_B D_A) V_C =$$

$$= -T_{AB}{}^Q D_Q V_C - \mathcal{R}_{ABC}{}^D V_D - (\mathcal{F}_{AB} V_C - (-1)^{c(a+b)} V_C \mathcal{F}_{AB}), \qquad (14)$$

where V_C is a vector superfield, $\mathcal{F}_{AB}$ is a gauge field which is in the algebra of internal symmetry group G, the supercurvature $\mathcal{R}_{ABCD}$ differs in sign in comparison with [20]. We introduce "by hands" the dilatino superfield ϕ and use:

3) The most general representation for spinorial derivative of the dilatino χ-superfield ($\chi_\alpha \equiv D_\alpha \phi$):

$$D_\alpha \chi_\beta = -\frac{1}{2} \Gamma^f_{\alpha\beta} D_f \phi + (-\frac{1}{36} \phi T_{abc} + \alpha' A_{abc}) \Gamma^{abc}_{\alpha\beta}, \qquad (15)$$

where A_{abc} is an arbitrary completely antisymmetric superfield, which is determined later in terms of torsion and curvature, when the axionic superfield Bianchi Identities will be considered. It will be clear later, that the A_{abc}-field contribution as defined in (15) is really proportional to α'.

The complete set of additional constraints and equations of motion for superfields of the supergravity multiplet were derived from (12)-(15) in [20], see also [19]. They are presented in Appendix 2. These equations are transformed into equations for usual fields, if one calculates spinorial derivatives of A_{abc}-superfield in terms of torsion and curvature (see below), and takes zero superspace components. (In the following we use the same notations for physical fields and corresponding superfields in the cases when it can not lead to a confusion).

3 Gauge Sector

The derivation of gauge matter fields equations is the standard procedure in the superspace approach. (For example see [9], [22] and references therein). We present here some basic results (see [23]).

The Bianchi Identities for the gauge superfield $\mathcal{F}_{AB}$ are:

$$D_{[A}\mathcal{F}_{BC)} + T_{[AB}{}^Q \mathcal{F}_{|Q|C)} \equiv 0, \qquad (16)$$

where $\mathcal{F}_{AB} \equiv \mathcal{F}^J_{AB} X^J$, where $(X^J)_i{}^j$ are anti-hermitean matrices - generators of G.

We use the different notations $\mathcal{F}_{ab}$ and F_{ab} for the supercovariant and usual tangent-space components. (The connection see below). To solve (16) on the mass-shell the following constraint is needed:

$$\mathcal{F}_{\alpha\beta} = 0 \qquad (17)$$

Then, one can derive equation of motion in the form:

$$\Gamma^a D_a \lambda + \frac{1}{12} T_{abc} \Gamma^{abc} \lambda = 0, \qquad (18)$$

$$D^a \mathcal{F}_{ab} + T_{ba} \Gamma^a \lambda + 2 \lambda \Gamma_b \lambda = 0. \qquad (19)$$

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where λ is a gaugino superfield:

$$\mathcal{F}_{b\alpha} \equiv (\Gamma_b \lambda)_\alpha \tag{20}$$

(We do not write spinorial indices explicitly in cases, where their position can be reconstructed unambiguously).

The spinorial derivatives of λ and $\mathcal{F}^{ab}$- superfields also follow from (16), (17) (see Appendix 3).

4 The 3-form Axionic-Field Sector

The superspace Bianchi Identities for the axionic field take the form:

$$D_{[A}H_{BCD)} + \frac{3}{2}T_{[AB}{}^Q H_{|Q|CD)} =$$

$$= 3\alpha' \left(tr\,[\mathcal{R}_{[AB}\mathcal{R}_{CD)}] - tr[\mathcal{F}_{[AB}\,\mathcal{F}_{CD)}]\right) \tag{21}$$

The constraint is:

$$H_{\alpha\beta\gamma} = 0 \tag{22}$$

The 3-form K -field, considered in the Intoduction is connected with the H-superfield by the relation:

$$K_{mnp} = E_p{}^C E_n{}^B E_m{}^A H_{ABC}| \tag{23}$$

Here $K_{mnp} \equiv e_p^c e_n^b e_m^a K_{abc}$.

The mass-shell solution of (21) which is compatible with (12) - (15) can be obtained using the standard procedure [10], [9], [24] (see also [22]). We find nonzero components of H_{ABC}-superfield in the form:

$$H_{\alpha\beta a} = \phi\,(\Gamma_a)_{\alpha\beta} + \alpha'U_{\alpha\beta a}\,, \tag{24}$$

$$H_{\alpha bc} = -(\Gamma_{bc}\,\chi)_\alpha + \alpha'U_{\alpha bc}, \tag{25}$$

$$H_{abc} = -\phi\,T_{abc} + \alpha'U_{abc} \tag{26}$$

The dilaton field ϕ is defined by eq. (24). Then the Bianchi Identity (21) defines how it "penetrates" in all the other H-field components. In particular, $\chi = D\phi$ in (25) as a consequence of Bianchi Identities, etc.

The $U_{\alpha\beta a}$- and $U_{\alpha ab}$-superfields do not contain the contribution from gauge fields. The $U_{\alpha\beta a}$ is determined in the following form (U_a denotes the matrix $U_{\alpha\beta a}$,):

$$U_a = \left(-\frac{14}{9}T_{ab}^2 + \frac{2}{27}\eta_{ab}\,T^2\right)\Gamma^b +$$

$$+ \left(-\frac{2}{27}T_{aij}T_{klm} - \frac{1}{9}\eta_{ai}D_jT_{klm} - \frac{1}{9}\eta_{ai}T^2{}_{jklm}\right)\Gamma^{ijklm} \tag{27}$$

The term $\sim \Gamma_a$ in U_a does not fixed by Bianchi Identities (BI's). As it follows from (24), the redefinition of such a term leads to the redefinition of the dilatonic field. Our choice in (27) is different from that in [17], namely: $\phi\,|_{ref\,[17]} \to (\phi + (2/27)T^2)$. That leads to simplification of final results.

Any contribution of the form $\sim \theta_{aijklm}\Gamma^{ijklm}$, where θ_{aijklm} is a comletely antisymmetric tensor, does not fixed by BI for the $H_{a\alpha\beta}$. Such a contribution is defined unambiguosly by BI for the $H_{\alpha bc}$.

The $U_{\alpha ab}$-field is equal to (U_{ab} denotes the $U_{\alpha ab}$):

$$U_{ab} = 4\Gamma_{i[a}D^i L_{b]} + \frac{2}{3}L^i T_{abi} + \left(\frac{7}{18}\Gamma^{ijk}{}_{[a}L_{b]} - \right.$$

$$\left. -\frac{1}{3}\Gamma_{ab}{}^{ij}L_k + \frac{2}{3}\Gamma^{ij}{}_{[a}T_{b]}{}^k + \frac{2}{9}\Gamma^{ijk}T_{ab} - \frac{2}{3}\Gamma_{ab}{}^i T^{jk}\right) T_{ijk} + $$

$$+ \left(-\frac{1}{2}\Gamma^{ij}L_{[a} - \frac{4}{3}\Gamma_{[a}{}^i L_j - \frac{20}{3}\Gamma^i T^j{}_{[a} + \frac{8}{3}\Gamma_{[a}T^{ij}\right) T_{b]ij} \qquad (28)$$

The U_{abc}-field is equal to:

$$U_{abc} = U_{abc}^{(grav)} + U_{abc}^{(gauge)} \qquad (29)$$

where the gauge-field contribution is:

$$U_{abc}^{(gauge)} = -tr(\lambda\,\Gamma_{abc}\,\lambda) \qquad (30)$$

but the gravity-field contribution is:

$$U_{abc}^{(grav)} = -2\,D^2 T_{abc} - 6\,D^i T^2{}_{i[abc]} - 6\,T^{ij}{}_{[a}D_b T_{c]ij} - 6\,\mathcal{R}^{ij}{}_{[ab}T_{c]ij} - $$

$$-6\,\mathcal{R}_{i[a}T_{bc]}{}^i + 4\,T^3_{[abc]} - T_{ij}\Gamma_{abc}T^{ij} - 12\,T_{i[a}\Gamma_b T_{c]}{}^i - $$

$$- 6\,T_{[ab}L_{c]} - L_i\Gamma_{abc}L^i - 12\,L_{[a}\Gamma_b L_{c]} \qquad (31)$$

These superfields were discussed earlier in [9], [10], [24] using another parametrization (another set of constraints).

The A-superfield in (2.4) is also defined unambiguously from the (2.2)-component of the Bianchi Identity (21) (the (p,q)-component of a superform contains p bosonic and q fermionic indices). We get:

$$A_{abc} = A_{abc}^{(grav)} + A_{abc}^{(gauge)} \qquad (32)$$

where

$$A_{abc}^{(gauge)} = -\frac{1}{24}\,tr(\lambda\,\Gamma_{abc}\,\lambda) \qquad (33)$$

and

$$A_{abc}^{(grav)} = -\frac{1}{18}D^2 T_{abc} + \frac{1}{36}D^i T^2{}_{i[abc]} - \frac{1}{36}T^{ij}{}_{[a}D_b T_{c]ij} - \frac{5}{1944}T^2 T_{abc} - $$

$$-\frac{5}{108}T^2{}_{i[a}T_{bc]}{}^i + \frac{5}{54}T^3_{[abc]} - \frac{1}{3888}\epsilon_{abc}{}^{ijklmnp}T_{ijk}\left(D_l T_{mnp} + \frac{5}{2}T^2{}_{lmnp}\right)$$

$$-\frac{1}{24}T_{ij}\Gamma_{abc}T^{ij} - \frac{1}{48}L_i\Gamma_{abc}L^i + \frac{1}{2}L_{[a}\Gamma_b L_{c]} \qquad (34)$$

The A_{abc}-superfield is a solution of eq.'s (A2.9), (A2.10'). That provides a good check of the result.

Now we are ready to discuss equations (A2.1)-(A2.6) in terms of fields from the supergravity multiplet. One must use for this purpose the expression of T_{abc} in terms of H_{abc}-field. This expression follows from (26) as a perturbative series in α'. By this way one obtains equations of motion as a series in α'. All the spinorial derivatives from the A_{abc}-field can be calculated in terms of fields of supergravity multiplet in the desired (zero) order in α' using exact relations from Appendix 3. The supersymmetry transformations also presented as a series in α' in this case. The lowest $\sim (\alpha')^0$-order corresponds to the supergravity by Chapline-Manton [8]. The next $\sim \alpha'$-order was considered explicitly at the mass-shell by Pesando [11] using results from [9], [10]. The lagrangian in the α' order has not been constructed. (Unfortunately, because of differences in parametrization and some differences in the approach we are not able to use intermediate results from [11]).

Calculations in the highest orders become tremendously cumbersome. But, it seems inconsistent to consider terms $\sim (\alpha')^p \quad p \geq 2$ because terms $\sim (\alpha')^2$ of σ-model loop expansion were not taken into account.

We don't consider approximate equations of motion for standard supergravity in the $\sim \alpha'$-order, because this program will be realized exactly (without any expansion in α') for the dual supergravity in the next section . Then it will be a simple algebraic problem to come back to the usual Type I supergravity case, making the dual transformation (see below).

In spite of a complicated structure of H-field equations that follow from (21), (22), one can make a useful check of the procedure. Note, that eq. (A2.5) must be interpreted as the H-field Bianchi Identity. So it must coincide with the (4,0)- component of (21). We have checked that is really the case.

Namely, one can easily prove, that the difference between (A2.5) and the (4,0) component of (21) is equal to the (4,0)-component of the superform identity [9]:

$$D\,U^{(grav)} + V = tr R^2 \qquad (35)$$

where $U_{(0.3)}^{(g)} = V_{(0.4)} = V_{(1.3)} = 0$. The components $V_{(2,2)}$, $V_{(3,1)}$ can be easily calculated from (21). Equation (35) is identically satisfied for (2,2), (1,3), (0,4) components because it is reduced exactly to that used for *definition* of A and $U^{(grav)}$-superfields. Then equations corresponding to (4.0), (3.1)-components follow identically by algebraic manipulations from the equations corresponding to (2,2), (1,3), (0,4)- components (cf. [9]).

One more remark is helpful for the following. All the relations in the theory under consideration are invariant under the scale transformation [25], [4]:

$$X_j \to \mu^{q_j}\, X_j \qquad (36)$$

where X_j is an arbitrary field, q_j is a numerical factor, μ is an arbitrary common factor. It is a classical symmetry, because the lagrangian is also transformed according to (36) with $q \neq 0$.

We present in the Table 1 the transformation rules for different fields (the numerical factors in the table are the values of q_j for each field):

Table 1

ϕ	-1	T_{abc}	$-1/2$	T_{ab}^{γ}	$-3/4$
e_m^a	$1/2$	H_{abc}	$-3/2$	ψ_a^{γ}	$-1/4$
D_a	$-1/2$	N_{abc}	$-1/2$	χ	$-5/4$
D_α	$-1/4$	A_{abc}	$-3/2$	$R_{ab}{}^{cd}$	-1
$\mathcal{F}_{ab}$	-1	λ	$-3/4$	$\mathcal{L}$	-2

Now we come to consideration of dual supergravity.

5 The 7-form Axionic-Field Sector

One can interpret the same equations (A2.1)-(A2.6) in terms of the 7-form graviphoton superfield $N_{A_1...A_7}$. The Bianchi Identity for such a field takes the form:

$$D_{[A_1} N_{A_2...A_8)} + \frac{7}{2} T_{[A_1A_2}{}^Q N_{|Q|A_3...A_8)} \equiv 0 \tag{37}$$

We don't introduce the term $\sim \beta'(DX_7)$ in the r.h.s. of eq.(37) according to the discussion in Introduction. (Note, that such a term breaks the scale invariance (36)). It is an additional indication that contribution $\sim \beta'$ corresponds to loop corrections in the usual supergravity). The following nonzero components provide the mass-shell solution of (37) which is consistent with (12)-(15):

$$N_{\alpha\beta a_1...a_5} = -(\Gamma_{a_1...a_5})_{\alpha\beta}, \tag{38}$$

$$N_{abc} = T_{abc}, \tag{39}$$

where N_{abc} is defined by:

$$N_{abc} \equiv \frac{1}{7!} \varepsilon_{abc}{}^{a_1...a_7} N_{a_1...a_7} \tag{40}$$

Note the connection between the supercovarian N-field and the M-field discussed in the Introduction:

$$M_{n_1...n_7} = E_{n_7}{}^{A_7} ... E_{n_1}{}^{A_1} N_{A_1...A_7}| \tag{41}$$

Here $M_{n_1...n_7} \equiv e_{n_7}^{a_7} ... e_{n_1}^{a_1} M_{a_1...a_7}$.

It is important, that the solution (38), (39) is valid for *any* A_{abc}-field, in particular for that, derived in usual supergravity (see eq. (32)).

Using (39) in the equations of Appendix 2 and defining the A-field according to (32), with the substitution (39), we get the mass-shell description of dual supergravity in a closed and relatively simple form (as opposed to the usual supergravity case!).

Using (39) in the eq. (26) we get the duality relation between the H_{abc} and $N_{a_1...a_7}$-superfields:

$$H_{abc} = -\phi N_{abc} + \alpha' U_{abc}|_{T_{ijk} \to N_{ijk}} \tag{42}$$

where U_{abc} is defined in (29)-(31).

Now we come to the detailed study of equations of motion and to the lagrangian construction in the dual supergravity.

6 Lagrangian for Gauge Fields

One must change variables in eq.'s (18),(19) from the supercovariant to usual one's with help of the relations (A1.1)-(A1.10). In particular, the relation (A1.10) is important (it follows immediately from (39) and definition of the $\tilde{M}_{abc}$-field in eq.(A1.5) and (41)). The resulting equations take the lagrangian form and the corresponding lagrangian is equal to (compare with [26], [15]):

$$
e^{-1}\,\mathcal{L}^{(gauge)} = \frac{1}{g^2}\,tr\,\left[\frac{1}{4}\,F_{ba}\,F^{ba} - \frac{1}{8\cdot 6!}\,\varepsilon^{a_1...a_{10}}\,C_{a_1...a_6}\,F_{a_7 a_8}\,F_{a_9 a_{10}} - \right.
$$

$$
-\lambda\,\hat{\nabla}\,\lambda + \frac{1}{24}\,\lambda\,\hat{\tilde{M}}\,\lambda - \frac{1}{2}\,\lambda\,\Gamma^a\,\hat{F}\,\psi_a -
$$

$$
\left. -(\lambda\,\Gamma_b\,\psi_a)\,(\lambda\,\Gamma^a\,\psi^b) + \frac{1}{2}\,(\lambda\,\Gamma^b\,\psi_b)^2 + \frac{1}{2}\,(\lambda\,\Gamma_a\,\psi_b)^2\right], \tag{43}
$$

Here the gauge-field coupling constant g is introduced. Eq. 43 disagrees with [26] in some terms of fourth order in fermions.

To find the value of g^2 in terms of α', one must consider the gauge-field contribution in the supergravity-multiplet equations of motion presented in (A2.1)-(A2.6). This contribution resulted from the $A_{abc}^{(gauge)}$-superfield (see (33) and it's spinorial derivatives). Just the same contribution one must obtain, making the variation of $\mathcal{L}^{(gauge)}$ over the fields of gravity multiplet. The comparison of these contributions makes it possible to find g^2 and to establish linear combinations of equations (see below relations (48) -(50)) that follow from the lagrangian. It is sufficient to put $A_{abc}^{(grav)} = 0$ at this stage. We get:

$$
\alpha' = -\frac{1}{4\,g^2} \tag{44}
$$

This relation follows from the consideration of gauge matter contribution to the graviton and (independently) gravitino equations of motion.

7 Zero Order Lagrangian for Gravity

It is instructive now, as a first step, to discuss the gravity-part of a total lagrangian in the limit $\alpha' \to 0$ starting from equations (A2.1) - (A2.6). It is possible to write this lagrangian in a simple form [27], which follows from the linearity in ϕ and χ -fields of the equations from Appendix 2:

$$
e^{-1}\,\mathcal{L}_0^{(grav)} = \phi\,(\mathcal{R} - \frac{1}{3}\,T^2)\,|+ 2\chi\,\Gamma^{ab}T_{ab}\,| \tag{45}
$$

The symbol $|$ means as usual the zero superspace-component of a superfield. The complete explicit result for $\mathcal{L}_0^{(grav)}$ as derived in [27] takes the form:

$$
e^{-1}\,\mathcal{L}_0^{(grav)} = \phi\,R - \frac{1}{12}\,\phi\,\tilde{M}_{abc}^2 - 2\,\phi_{;\,a}\psi^a\Gamma_b\psi^b + 4\,\psi_a\Gamma^{ab}\chi_{;\,b} +
$$

$$-2\,\phi\,\psi_a\Gamma^{abc}\psi_{c;\,b} + \frac{1}{12}\,\phi\,\psi_a\Gamma^{[a}\hat{\tilde{M}}\Gamma^{b]}\psi_b - \frac{1}{2}\,\chi\,\Gamma^{ab}\psi^c\tilde{M}_{abc}-$$

$$-\frac{1}{48}\,\phi\,(\psi^d\Gamma_{dabcf}\psi^f)^2 + \frac{1}{4}\,\phi\,(\psi_a\Gamma_b\psi_c)^2 + \frac{1}{2}\,\phi\,(\psi^a\Gamma^b\psi^c)(\psi_a\Gamma_c\psi_b)-$$

$$-\,\phi\,(\psi_a\Gamma^b\psi_b)^2 + (\chi\Gamma_{ab}\psi_c)(\psi^a\Gamma^c\psi^b) - 2\,(\chi\Gamma_a\Gamma_b\psi^b)(\psi^a\Gamma_c\psi^c) \tag{46}$$

Up to field redefinitions it is the lagrangian obtained in [28], [4], [26]. Now we are ready to establish the relation between standard variables (see Introduction) and that used in the superspace approach. In particular: $\phi = \exp(2\varphi/3)$, $k^2 = 1/2$. The change of variables to the primed ones, that transforms (46) to the canonical form, is defined by:

$$e^a_m = \phi^{-1/8}\,e^a_m{}'$$

$$\chi = -\frac{2}{3\sqrt{2}}\phi^{17/16}\,\chi' \quad \psi_m = \frac{1}{2}\,\phi^{-1/16}\,(\psi_m{}' - \frac{1}{6\sqrt{2}}\,\Gamma_m{}'\,\chi') \quad etc. \tag{47}$$

The variation of $\mathcal{L}^{(gauge)} + \mathcal{L}^{(grav)}_0$ with respect to the gravitino field ψ_m produces the equation (see Appendix 2 for notations):

$$Q_a + \Gamma_a\,Q = 0. \tag{48}$$

The variation of the same object with respect to the axion field $C_{m_1\ldots m_6}$ produces the equation:

$$S_{abcd} + 3\,\psi_{[a}\,\Gamma_{bc}\,Q_{d]} = 0. \tag{49}$$

The variation with respect to the graviton field e^a_m produces the equation:

$$S_{ab} + \eta_{ab}\,(\frac{1}{2}B - S) - 2\psi_{(a}Q_{b)} - \frac{1}{2}\psi^c\Gamma_{ab}\,Q_c - \psi^c\Gamma_{c(a}Q_{b)}-$$

$$-\frac{1}{2}(\psi^c\Gamma_{cab} + 2\psi_a\Gamma_b)Q - \eta_{ab}\,\psi_c\Gamma^c Q - \frac{1}{2}\,\eta_{ab}\,Tr(\lambda\Lambda) - \frac{1}{4}\,Tr(\lambda\Gamma_{ab}\Lambda) = 0. \tag{50}$$

Here $B \equiv -\phi(\mathcal{R} - \frac{1}{3}T^2)$, but $\Lambda \equiv (\hat{\nabla}\lambda + \ldots) = 0$ is the l.h.s. of the gaugino equation (18). The variation with respect to the dilaton ϕ and the dilatino χ-fields produces the constraints (A2.7), (A2.8).

Calculating Γ_a projection from (48) one immediately obtains $Q = 0$, and then $Q_a = 0$. So, $S_{abcd} = 0$ as it follows from (49). Contracting a, b indices in (50) one obtains $S = 0$, and then $S_{ab} = 0$. So, all the equations (A2.1)-(A2.5) follow from (48)-(50). (Equation (A2.6) is equivalent to the Bianchi Identity for the M-field).

Now we come to consideration of α' contributions in pure gravity sector and to the construction of total lagrangian.

8 Total Lagrangian

The supersymmetric lagrangian of dual supergravity takes the form

$$\mathcal{L} = \mathcal{L}^{(gauge)} + \mathcal{L}^{(grav)} \tag{51}$$

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where:

$$\mathcal{L}^{(grav)} = \mathcal{L}_0^{(grav)} + \alpha' \mathcal{L}_1^{(grav)} \tag{52}$$

Now we are interested in the last term in (52). It is a property of our parametrization that $\mathcal{L}_1^{(grav)}$ does not depend on ϕ and χ fields. It means that the scale invariance (36) greatly simplify the possible structure of this term.

We consider the bosonic part of $\mathcal{L}_1^{(grav)}$ which contain 12 possible terms [6]:

$$\mathcal{L}_1^{(grav)} = \sum_{i=1}^{12} x_i \, L_i + \text{fermions} \tag{53}$$

where x_i are numbers to be determined by comparison with equations (A2.1)-(A2.6), but L_i are presented in the Table 2.

Table 2

i	L_i	i	L_i	i	L_i
1	R^2	5	$(\tilde{M}^2)\,R$	9	$\tilde{M}^{abc;d}(\tilde{M}^2)_{abcd}$
2	R_{ab}^2	6	$(\tilde{M}^2)_{ab}\,R^{ab}$	10	$(\tilde{M}^2)^2$
3	R_{abcd}^2	7	$(\tilde{M}^2)_{abcd}R^{abcd}$	11	$(\tilde{M}^2)_{ab}^2$
4	$\varepsilon^{0...9}R_{01bc}R_{23}{}^{bc}C_{4...9}$	8	$\tilde{M}^{abc}\nabla_d\nabla^d\tilde{M}^{abc}$	12	$(\tilde{M}^2)_{abcd}\,(\tilde{M}^2)^{acbd}$

where $(\tilde{M}^2)_{ab} = \tilde{M}_a^{cd}\tilde{M}_{bcd}$ and $(\tilde{M}^2)_{abcd} = \tilde{M}_{ab}^f\tilde{M}_{cdf}$.

Now we come to the determination of x_i in (53). All the terms, containing $\tilde{M}_{abc}$-field can be reconstructed with the help of the following simple procedure. As was discussed before, eq. $S_{abcd} = 0$ is equivalent to the (4,0)-component of the H-field Bianch Identity, which (dropping spinorial terms) takes the form:

$$D_{[a}H_{bcd]} + \frac{3}{2}\,T_{[ab}{}^f\,H_{|f|cd]} =$$

$$= 3\,\alpha'\,\left(\mathcal{R}_{[ab}{}^{ef}\mathcal{R}_{cd]\,fe} - tr[\mathcal{F}_{[ab}\,\mathcal{F}_{cd]}]\right) \tag{54}$$

Changing notations to the covariant derivative ∇_a, to the standard curvature R_{abcd} an to the gauge-field F_{ab}, one can write (54) in the form:

$$\left(H_{[abc} - 3\,T_{ij[a}R_{bc}{}^{ij} + \frac{3}{2}T_{ij[a;b}T_c{}^{ij} + \frac{1}{2}T_{ij[a}(T^2)_{bc}{}^{ij}\right)_{;d]} =$$

$$= 3\,\alpha'\,\left(-R_{[ab}{}^{ij}R_{cd]ji} + tr\,[F_{[ab}\,F_{cd]}]\right) \tag{55}$$

Here $;d$ denotes the covariant derivative ∇_d.

Then one can use (42) and relations from Appendix 1, writing everything in terms of $\tilde{M}_{abc}$-field. After that the terms in the lagrangian, containing the $\tilde{M}_{abc}$-field, are immediately reproduced from the l.h.s. of (55) which has the desired form of complete derivative. The terms $\sim MR^2$ and $\sim MF^2$ are reproduced from the r.h.s. of (55).

To check the result and to obtain another terms in the lagrangian, one needs the explicit form of equations in Appendix 2. So, one must calculate the first and second spinorial derivatives from the A_{abc}-field. We have done this calculation. See the result

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[6]K.N.Zyablyuk, unpublished

in Appendix 4. We are able to present explicitly the dilaton equation (in the complete form) and the graviton equation with bosonic field contributions only.

Such a calculation is possible only with the help of a computer. We use the program "GRAMA" written by us in "MATHEMATICA" environment which makes it possible to perform calculations effectively using the PC-486 with RAM=16 Mb.

One can obtain terms, containing $\tilde{M}$-field and other terms $\sim R^2$ in (53) by the following way. Calculating the variation of $\mathcal{L}^{(grav)}$ over the graviton field one must get the equation (50). Contracting indices a, b one gets the dilaton equation $S = 0$, as it is explained in Appendix 2. Comparing the result with the equation (A4.1) one finds values of x_i in eq. (53). The result of this calculation is in complete correspondence with the previous one, based on eq. (55). The obtained values of x_j are presented in the Table 3.

Table 3

x_1	$undetermined$	x_7	0
x_2	2	x_8	$-1/6$
x_3	-1	x_9	$1/2$
x_4	$(2 \cdot 6!)^{-1}$	x_{10}	$x_1/144$
x_5	$-2\,x_1/12$	x_{11}	0
x_6	$-1/2$	x_{12}	$-1/24$

Then we ensure, that graviton equation (A4.2) follows from the total lagrangian (51). That provides a complete check of the result.

Terms containing x_1 in (53) enter in the combination $x_1\,(R - \frac{1}{12}M^2)^2$ which is the square of the constraint (A2.8) (remind that we are working in the zero order in fermionic fields, remind also the difference between R and $\mathcal{R}$ fields). This constraint is satisfied automatically on the mass-shell. That is the reason why x_1 is undetermined. One can put x_1 equal to zero without any effect on equations of motion. There is another argument to omit terms $\sim x_1$: one can cancel all such terms by the off-shell ϕ-field redefinition: $\phi \rightarrow \phi - x_1 \cdot \alpha'\,(R - \frac{1}{12}\tilde{M}^2)$.

Finally, (puting $x_1 = 0$) one can write the bosonic part of the gravity lagrangian (52) in the form:

$$\mathcal{L}^{(grav)} = \phi\,(R - \frac{1}{12}\tilde{M}^2)+$$

$$+\alpha'\left[2\,R_{ab}^2 - R_{abcd}^2 + \frac{1}{2 \cdot 6!}\varepsilon^{abcdf_1...f_6}\,R_{ab}{}^{ij}R_{cdij}C_{f_1...f_6} - \frac{1}{2}\,R^{ab}(\tilde{M}^2)_{ab}-\right.$$

$$\left.-\frac{1}{6}\,\tilde{M}^{abc}\nabla_f\nabla^f\tilde{M}_{abc} + \frac{1}{2}\,\tilde{M}^{abc;d}(\tilde{M}^2)_{abcd} - \frac{1}{24}\,(\tilde{M}^2)_{abcd}(\tilde{M}^2)^{acbd}\right] + \text{spinors} \qquad (56)$$

One can restore the ghost-free Gauss-Bonnet combination [29]: $R_{abcd}^2 - 4\,R_{ab}^2 + R^2$, adding the square of the constraint (A2.8) and the square of the graviton equation (A2.4) to the lagrangian (56). It does not change equations of motion but makes the lagrangian much more cumbersome. It is one of the reasons why attempts to supersymmetrize the Gauss-Bonnet combination started in [13] were not succesful.

One can make by a standard procedure the dual transformation in the total lagrangian (51), adding the term with the lagrangian multiplier B_{mn}:

$$\Delta\mathcal{L} = \frac{1}{2}B_{mn}\partial_n\tilde{M}^{mnp} = -\frac{1}{6}\left[K_{mnp} - 2\,\alpha'(\Omega_G - \Omega_L)_{mnp}\right]\tilde{M}^{mnp} \qquad (57)$$

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After that, one can consider $\tilde{M}_{abc}$ as an independent variable. Solving the equation of motion for $\tilde{M}_{abc}$, one is able to reproduce the bosonic part of dual transformation (42) with $K_{abc} - 2\,\alpha'\,X_{abc}$ instead of H_{abc} in the l.h.s. of (42). Here X_{mnp} is a 3-form field defined in (A1.13). The term $2\alpha'X_{abc}$ describes the change of the CS-term in K_{mnp} to that defined by torsion-full spin-connection. So, there is a complete correspondence between (51) and dual transformation (42). Using (42) one can rewrite the lagrangian in terms of K_{abc}-field, obtaining the lagrangian for standard formulation of supergravity. But in this case the result can be presented as an infinite series in $(\alpha')^p, \ p \geq 1$.

Appendix 1

Notations

The 10-dimensional metric signature is: $\eta_{ab} = diag(1, -1, \ldots -1)$. The tangent-space vector indices are from the beginning of the alphabet: $a, b, \ldots$; the world indices are from the middle of the alphabet: $m, n, \ldots$. We use the 16-components spinors and spinorial indices are $\alpha, \beta, \ldots$. Gamma-matrices are $\Gamma^a_{\alpha\beta}, (\Gamma^{ab})_\alpha{}^\beta$, etc. The algebraic properties of Γ-matrices are described in many papers. We use extensively relations from [9], [21]. The superspace indices are $M = (m, \mu), \ N = (n, \nu), \ldots$ and $A = (a, \alpha), \ B = (b, \beta), \ldots$.

We use standard conventions: $tr\, F \wedge F = d\Omega_G, \ tr\, R \wedge R = d\Omega_L$, where symbol tr means *trace* in the vectorial representation of the corresponding group, i.e. the $O(1,9)$-group for Ω_L and $SO(32)$ for Ω_G. One must change $tr \rightarrow (1/30)Tr$ for the gauge-group $E_8 \times E_8$, where Tr means *trace* in the adjoint representation. The same change is possible also for $SO(32)$. Usually we drop the $\wedge$-sign in products of forms.

The Lorentz Chern-Symons term is defined by: $\Omega_L = tr\,(\omega d\omega + 2\,\omega^3/3)$, where ω is the spin-connection 1-form, $R = d\omega + \omega^2$. The same expressions are used for the gauge-group CS-term Ω_G and the gauge-field F in terms of the 1-form potential A.

The following notations are used in (A2.1)-(A2.10) below and in the main text:

$$L_a = T_{ab}\Gamma^b, \quad \hat{Z} = Z_{ijk}\Gamma^{ijk}$$

$$YZ = Y_{ijk}Z^{ijk}, \quad (YZ)_{ab} = Y_{aij}Z_b{}^{ij}, \quad (YZ)_{abcd} = Y_{abj}Z_{cd}{}^j,$$

$$Z^3_{abc} = Z_{aij}Z_b{}^{jk}Z_{ck}{}^i$$

where Y, Z are 3-rd rank antisymmetric tensors.

We present also relations between supercovariant and space-time covariant objects. The gravitino field ψ_b^α is defined with the help of the superspace veilbein (cf. [16]):

$$E_M{}^A| = \begin{pmatrix} e_m{}^a & \psi_m^\alpha \\ 0 & \delta_\mu^\alpha \end{pmatrix}, \tag{A1.1}$$

The supercovariant derivative $D_a \equiv E_a{}^M D_M$ is equal to:

$$D_a = e_a^m D_m - \psi_a^\beta D_\beta, \tag{A1.2}$$

where $\psi_a = e_a^m \psi_m$, the space-time component of the covariant derivative is:

$$D_m\lambda = \partial\,\lambda - \phi_m\,\lambda - [A_m, \lambda], \tag{A1.3}$$

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where $(\phi_m)^\beta{}_\gamma \equiv \frac{1}{4}\phi_m{}^{ab}(\Gamma_{ab})^\beta{}_\gamma$ is the spin-connection which is in the algebra of $O(1.9)$.

We introduce also the usual tangent-space components of physical fields instead of supercovariant quantities. (Note, that supercovariant components are equal to: $\mathcal{F}_{ab} = E_a^M E_b^N \mathcal{F}_{MN}$, etc.). Namely:

$$F_{ab} \equiv e_a^m e_b^n F_{mn}, \qquad \omega_{cab} \equiv e_c^m \omega_{mab}, \qquad (A1.4)$$

$$\tilde{M}_{abc} = \frac{1}{7!} \varepsilon_{abc}{}^{a_1...a_7} \left(e_{a_1}{}^{m_1} \ldots e_{a_7}{}^{m_7} M_{m_1...m_7}\right), \qquad (A1.5)$$

where $M_{m_1...m_7} = 7\,\partial_{[m_1} C_{m_2...m_7]}$, and $C_{m_1...m_6}$ is the 6-form axionic potential.

One finds the relation by a standard procedure between the torsion-full spin-connection in eq.(A1.3) and the usual spin-connection ω_{cab} defined in terms of e_m^a:

$$\phi_{cab} = \omega_{cab}(e) + \frac{1}{2} T_{cab} + S_{cab}, \qquad (A1.6)$$

where:

$$S_{cab} = \psi_a \, \Gamma_c \, \psi_b - \frac{3}{2}\psi_{[a} \, \Gamma_c \, \psi_{b]} \qquad (A1.7)$$

We use also the notation ∇_m for the covariant derivative with the spin-connection $\omega_m^{(0)}$ ($\nabla_{[m} e_{n]}^a = 0$), $\nabla_a \equiv e_a^m \nabla_m$.

To be complete we present the connection between physical fields introduced before and supercovariant fields (on the mass shell):

$$\mathcal{F}^{ab} = F^{ab} + 2\,\psi^{[a}\,\Gamma^{b]}\,\lambda \qquad (A1.8)$$

$$T_{ab} = 2\,\nabla_{[a}\,\psi_{b]} + \frac{1}{2}\left(\Gamma^{cd}\right)\psi_{[a}\,C_{b]cd}-$$

$$-\frac{1}{72}T_{cde}(\Gamma_{[a}\Gamma^{cde} + 3\Gamma^{cde}\Gamma_{[a})\psi_{b]} \qquad (A1.9)$$

$$T_{abc} = \tilde{M}_{abc} - \frac{1}{2}\,\psi_f\,\Gamma^f{}_{abc}{}^d\,\psi_d \qquad (A1.10)$$

$$\mathcal{R} - \frac{1}{3}T_{abc}^2 = R - \frac{1}{12}(\tilde{M}_{abc})^2 + \text{spinorial} \quad \text{terms.} \qquad (A1.11)$$

$$\mathcal{R}_{mnab} = R_{mnab} + \nabla_{[m}\,T_{n]ab} - \frac{1}{2}T_{a[mn]b}^2 + \text{spinorial terms} \qquad (A1.12)$$

In deriving of (A1.10) relations (39), (A1.5) were used. $\mathcal{R}$ is defined in terms of spin-connection ϕ, but R -in terms of $\omega(e)$

It is instructive to present the relation between the CS-term $\tilde{\Omega} = tr\,(\phi d\phi + 2\phi^3/3)$ and usual CS-term, defined in terms of $\omega_{mab}(e)$. One gets (in form notations):

$$\tilde{\Omega} = \Omega + X + \text{spinors}, \quad X = \frac{1}{12}T^3 + TR + \frac{1}{4}TdT + \frac{1}{4}d(\omega T) \qquad (A1.13)$$

where one-form field $(T_m)_a{}^b \equiv T_{mac}\eta^{cb}$ is introduced.

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Appendix 2

Constraints and equations of motion

We present here equations of motion and constraints that follow from the mass-shell solution of Bianchi Identities.

Gravitino equation of motion:

$$Q_a \equiv \phi L_a - D_a \chi - \frac{1}{36} \Gamma_a \hat{T} \chi - \frac{1}{24} \hat{T} \Gamma_a \chi + \alpha' \left(\frac{1}{42} \Gamma_a \Gamma^{ijk} D A_{ijk} + \frac{1}{7} \Gamma^{ijk} \Gamma_a D A_{ijk} \right) = 0, \quad (A2.1)$$

Dilatino equation of motion:

$$Q \equiv \hat{D} \chi + \frac{1}{9} \hat{T} \chi + \frac{\alpha'}{3} \Gamma^{ijk} D A_{ijk} = 0. \qquad (A2.2)$$

Dilaton equation of motion:

$$S \equiv D_a^2 \phi + \frac{1}{18} \phi T^2 - \alpha' \left(2 TA + \frac{1}{24} D \Gamma^{ijk} D A_{ijk} \right) = 0. \qquad (A2.3)$$

Graviton equation of motion:

$$S_{ab} \equiv \phi \mathcal{R}_{ab} - L_{(a} \Gamma_{b)} \chi - \frac{1}{36} \phi \eta_{ab} T^2 + D_{(a} D_{b)} \phi +$$

$$+ \alpha' \left(-2 T_{(a} A_{b)} + \frac{3}{28} D \Gamma^{ij}{}_{(a} D A_{b) ij} - \frac{5}{336} \eta_{ab} D \Gamma^{ijk} D A_{ijk} \right) = 0. \qquad (A2.4)$$

Axionic equations of motion and Bianchi Identities:

$$S_{abcd} \equiv D_{[a} (\phi T_{bcd]}) + \frac{3}{2} T_{[ab} \Gamma_{cd]} \chi + \frac{3}{2} \phi T^2_{[abcd]} +$$

$$+ \alpha' \left(\frac{1}{12} (T \epsilon A)_{abcd} + 6 \, (TA)_{[abcd]} + \frac{3}{4} D \Gamma_{[ab}{}^j D A_{cd]j} \right) = 0. \qquad (A2.5)$$

$$D^a T_{abc} = 0, \qquad (A2.6)$$

There are constraints:

$$T_{ab} \Gamma^{ab} = 0, \qquad (A2.7)$$

$$\mathcal{R} - \frac{1}{3} T^2 = 0, \qquad (A2.8)$$

where $\mathcal{R}$ is a supercurvature scalar ($\mathcal{R} \equiv \mathcal{R}_{abcd} \eta^{ac} \eta^{bd}, \quad T^2 \equiv T_{abc} T^{abc}$). (There are additional relations, which are not interesting for our purposes here, see [20] for details).

Note, that

$$\Gamma^a Q_a = -Q, \quad \eta^{ab} S_{ab} = S$$

The components of the supercurvature are defined from (12), (13) in the form:

$$\mathcal{R}_{\alpha\beta ab} = \frac{5}{6} T_{abc} \Gamma^c_{\alpha\beta} + \frac{1}{36} T_{ijk} (\Gamma^{ijk}{}_{ab})_{\alpha\beta},$$

$$\mathcal{R}_{abc} = 2\,T_{a[b}\Gamma_{c]} - \frac{3}{2}L_{[a}\Gamma_{bc]}$$

Furthemore:

$$\mathcal{R}_{[abc]d} = D_{[a}T_{bc]d} + T^2_{[abc]d}, \quad \mathcal{R}_{[ab]} = 0,$$

There are two equations for the superfield A_{abc}. The first one follows from the self-consistency of equations of motion (cf. [19], [20]):

$$D\Gamma_{[a}{}^{ij}D\,A_{b]ij} + 56\,D^j A_{jab} - \frac{64}{3}(TA)_{[ab]} = 0. \tag{A2.9}$$

The second one [20], [9] means, that 1200 IR contribution to the A-field spinorial derivative is equal to zero:

$$(D_\alpha A_{abc})^{(1200)} = 0, \tag{A2.10}$$

It follows from (A2.10):

$$DA = \Gamma_{abc}{}^{ij}X_{ij} \tag{A2.10$'$}$$

where X_{ij}^α is an arbitrary function which is 16+144+560 representation of $O(1.9)$. It follows also from (A2.10):

$$D_{[a}\,A_{bcd]} + 2\,(TA)_{[abcd]} + \frac{1}{360}(T\epsilon A)_{abcd} -$$

$$-\frac{1}{16\cdot 60}D\Gamma_{abcd}{}^{ijk}D\,A_{ijk} + \frac{1}{16}D\Gamma_{[ab}{}^i D\,A_{cd]i} = 0, \tag{A2.10$''$}$$

Appendix 3

Spinorial derivatives

We present here spinorial derivatives of fields entering into the A_{abc}-superfield in (33), (34). All the results, written below, follow from (12), (13), and (for gauge fields)- from (16), (17). The results for curvature tensor spinorial derivatives were checked by us independently using the curvature tensor Bianchi Identity:

$$D_{[A}\,\mathcal{R}_{BC)D}{}^E + T_{[AB}{}^Q\,\mathcal{R}_{|Q|C)D}{}^E = 0 \tag{A3.1}$$

Spinorial derivative of the curvature superfield is:

$$D\mathcal{R}_{abij} = 2\,D_{[a}\,\mathcal{R}_{b]ij} + \frac{1}{36}\,T^{mns}\,\Gamma_{mns}\Gamma_{[a}\,\mathcal{R}_{b]ij} +$$

$$+\mathcal{R}_{dij}\,T_{ab}{}^d - \left(\frac{5}{6}\,T_{ijk}\Gamma^k + \frac{1}{36}\,T_{mnp}\Gamma^{mnp}{}_{ij}\right)T_{ab} \tag{A3.2}$$

where:

$$\mathcal{R}_{abc} = 2\,\Gamma_{[b}\,T_{c]a} + \frac{3}{2}\Gamma_{[ab}\,L_{c]} \tag{A3.3}$$

Spinorial derivatives of the torsion superfield are:

$$D\,T_{abc} = 3\,\Gamma_{[a}T_{bc]} + 3\,\Gamma_{[ab}\,T_{c]} \tag{A3.4}$$

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$$D_\alpha (T_{ab})^\beta = (\hat{O}_{ab})^\beta{}_\alpha \tag{A3.5}$$

where:

$$\hat{O}_{ab} = -\frac{1}{36}\Gamma_{[a}\Gamma^{ijk}D_{b]}\,T_{ijk} + \frac{1}{36\cdot72}\,\Gamma_{[a}\Gamma^{mnp}\Gamma_{b]}\Gamma^{ijk}\,T_{mnp}T_{ijk} -$$

$$+\frac{1}{72}\,\Gamma^m\Gamma^{ijk}T_{abm}T_{ijk} - \frac{1}{4}\mathcal{R}_{abij}\Gamma^{ij} \tag{A3.6}$$

Spinorial derivatives of matter fields are:

$$D_\alpha\lambda^\beta = \frac{1}{4}\mathcal{F}_{ab}(\Gamma^{ab})_\alpha{}^\beta \tag{A3.7}$$

and:

$$D\,\mathcal{F}_{ab} = 2\Gamma_{[a}\,D_{b]}\,\lambda - T_{abc}\,\Gamma^c\,\lambda - \frac{1}{36}\,T^{ijk}\Gamma_{ijk}\Gamma_{ab}\,\lambda \tag{A3.8}$$

Appendix 4

Dilaton and graviton equations of motion

We present here in explicit form the dilaton equation and bosonic contribution to the graviton equation of motion as they follow from (A2.3) and (A2.4). The dilaton equation:

$$D^2\phi + \frac{1}{18}\phi\,T^2 - \frac{1}{3}\alpha'\,tr\,\mathcal{F}_{ab}^2 - \alpha'\left[-\frac{2}{3}(\mathcal{R}_{ab})^2 + \frac{1}{3}(\mathcal{R}_{abcd})^2 + \frac{1}{3}\mathcal{R}^{ab}T_{ab}^2 -\right.$$

$$-\frac{1}{3}\mathcal{R}^{abcd}T_{abcd}^2 + \frac{1}{9}T^{abc}D^2T_{abc} - \frac{1}{6}D^2T^2 + \frac{1}{3}D^aD^bT_{ab}^2 +$$

$$+\frac{4}{3}L^a\hat{D}L_a + \frac{8}{3}T^{ab}D_aL_b + T^{abc}\left(-\frac{4}{3}L_a\Gamma_bL_c + \frac{5}{54}L^d\Gamma_{abc}L_d -\right.$$

$$\left.\left.-\frac{1}{9}L^d\Gamma_{ab}T_{cd} - \frac{1}{54}T^{df}\Gamma_{abc}T_{df} + \frac{7}{3}T_{ab}L_c + \frac{2}{3}T^d{}_a\Gamma_bT_{cd}\right)\right] = 0 \tag{A4.1}$$

The graviton equation:

$$\phi\mathcal{R}_{ab} + D_{(a}D_{b)}\phi - \frac{1}{36}\eta_{ab}\,\phi T^2 - \alpha'\,tr\left(2\,\mathcal{F}_a{}^j\mathcal{F}_{bj} - \frac{1}{6}\eta_{ab}\mathcal{F}_{ij}\mathcal{F}^{ij}\right) -$$

$$-\alpha'\left[-4\,\mathcal{R}_a{}^c\mathcal{R}_{bc} + 2\,\mathcal{R}_{ijka}\mathcal{R}^{ijk}{}_b - 2\,D^2\mathcal{R}_{ab} -\right.$$

$$-\frac{1}{6}D_aT_{ijk}D_bT^{ijk} + D_iT_{jk(a}\left(D_{b)}T^{ijk} - \frac{1}{2}D^iT_{b)}{}^{jk} - D^kT_{b)}{}^{ij}\right) +$$

$$+T^2{}_{ijk(a}\left(2\,D_{b)}T^{ijk} - 4\,D^iT_{b)}{}^{jk} - 2\,D^kT_{b)}{}^{ij}\right) + 4\,T_a{}^{ij}T_b{}^{kl}T^2{}_{ikjl} -$$

$$-2\,T^2{}_a{}^i{}_b{}^j T^2{}_{ij} + \eta_{ab}\left(\frac{1}{3}(\mathcal{R}_{ij})^2 - \frac{1}{6}(\mathcal{R}_{ijkl})^2 - \frac{1}{6}\mathcal{R}^{ij}T^2{}_{ij} + \frac{1}{6}\mathcal{R}^{ijkl}T^2{}_{ijkl} -\right.$$

$$\left.\left.-\frac{1}{18}T^{ijk}D^2T_{ijk} - \frac{1}{6}D^iD^jT^2{}_{ij} + \frac{1}{12}D^2T^2\right)\right] + \text{fermions} = 0 \tag{A4.2}$$

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Appendix 5

Supersymmetry transformations

Supersymmetry transformations for any physical field follow immediately from the super-gauge transformation for the corresponding superfield [16]. We get for gauge matter multiplet:

$$\delta_Q(\epsilon)\,\lambda = \frac{1}{4}\,\mathcal{F}_{ab}\,\Gamma^{ab}\,\epsilon$$

$$\delta_Q(\epsilon)\,A_m = -\lambda\,\Gamma_m\,\epsilon, \qquad (A5.1)$$

where ϵ^α is a parameter, $\Gamma_m \equiv e_m^a\,\Gamma_a$.

For gravity multiplet :

$$\delta_Q(\epsilon)e_m{}^a = -\psi_m\Gamma^a\epsilon\,,$$

$$\delta_Q(\epsilon)\psi_m = -D_m\epsilon - \frac{1}{72}\,\Gamma_m\hat{T}\,\epsilon\,,$$

$$\delta_Q(\epsilon)\phi = \chi\,\epsilon\,,$$

$$\delta_Q(\epsilon)\chi = \frac{1}{2}\,\partial_m\phi\,\Gamma^m - \frac{1}{2}(\psi_m\,\chi)\,(\Gamma^m\epsilon) - (\frac{1}{36}\,\phi\hat{T} - \alpha'\hat{A})\,\epsilon\,,$$

$$\delta_Q(\epsilon)C_{m_1\ldots m_6} = 6\,\psi_{[m_1}\Gamma_{m_2\ldots m_6]}\,\epsilon \qquad (A5.2)$$

It is the advantage of our parametrization, that matter degrees of freedom as well as superstring $\sim \alpha'$ corrections "penetrate" the gravity multiplet supersymmetry transformations only due to the A_{abc}-contribution.

The supersymmetry algebra for physical fields is closed up to equations of motion and gauge transformations. Namely:

$$[\delta_Q(\epsilon_2),\,\delta_Q(\epsilon_1)]\,X = (\delta_{GCT}(\xi^m) + \delta_Q(\epsilon') + \delta_L(L_{ab}) +$$

$$+ \delta_G(\Omega_{YM}) + \delta_A(f_{n_1\ldots n_5}))\,X + \text{(equations of motion)}, \qquad (A5.3)$$

where X is any field from gravity or matter multiplet, δ_{GCT} is a general coordinate transformation, δ_L is a Lorentz transformation, δ_G is a matter gauge-field transformation, δ_A is an axion-field gauge transformation.

The transformation parameters are:

$$\xi^m = \epsilon_1\,\Gamma^m\,\epsilon_2\,.$$

$$\Omega_{YM} = -\xi^m\,A_m\,.$$

$$\Omega_{m_1,\ldots,m_5} = -\xi^n\,C_{m_1,\ldots,m_5,n}$$

$$L_{ab} = -\xi^n\,\phi_{nab} + \frac{5}{12}\,\xi^c\,T_{abc} + \frac{1}{36}\,\epsilon_1\,\Gamma_{ab}{}^{ijk}\,\epsilon_2\,T_{ijk}.$$

$$\epsilon' = \xi^n\,\psi_n$$

Eq. (A5.3) takes place for any A_{abc}-field, not specifically for that, defined by eq.(32). Only the representation (A2.10') for the A_{abc}-superfield spinorial derivative is necessary for the derivation of (A5.3) .

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